

Mathematics: analysis and approaches**Higher level****Paper 1**

Name

Date: _____

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written in the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (56 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the complex number $z = 1 + i$.

(a) Express z in modulus-argument form $r \operatorname{cis} \theta$. [3]

(b) Hence, find z^9 and express it in Cartesian form $a + bi$. [3]

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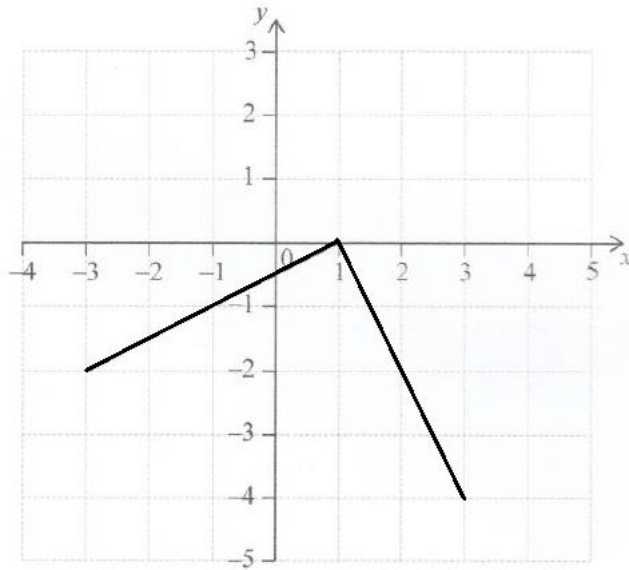
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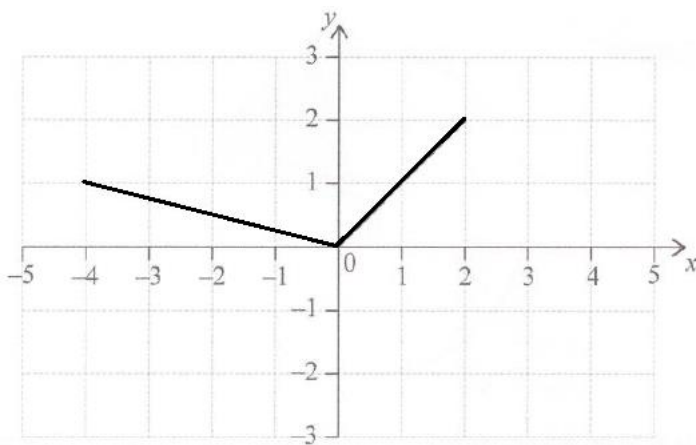
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2. [Maximum mark: 4]

Below is the graph of the function g for $-3 \leq x \leq 3$.



Another function, h , can be written in the form $h(x) = a[g(x+b)]$. The graph of h is shown below. Write down the value of a and the value of b .



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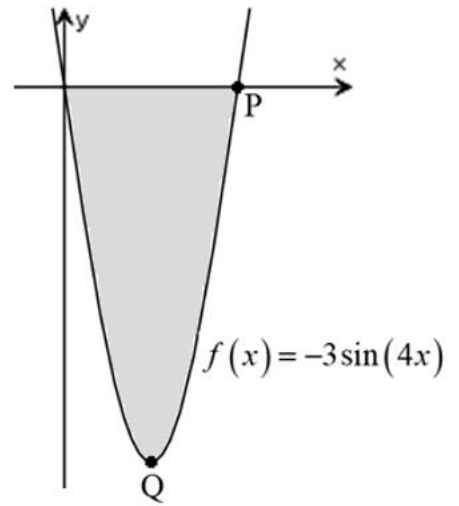
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3. [Maximum mark: 7]

A portion of the graph of $f(x) = -3\sin(4x)$ is shown.
 The point P is an x -intercept with coordinates $(p, 0)$.

- (a) Find the value of p . [2]
- (b) The point Q is a minimum. Write down the coordinates of Q. [2]
- (c) Find the area of the shaded region that is bounded by f and the x -axis. [3]



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4. [Maximum mark: 6]

The sum of the first three terms of an arithmetic sequence is 6 and the fourth term is 16.

Find the first term, u_1 , and the common difference, d , of the sequence.

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5. [Maximum mark: 7]

(a) Given $f(x) = x^2 + 4x - 10$, $x \leq -2$ show that $f^{-1}(x) = -2 - \sqrt{x+14}$, $x \geq -14$. [4]

(b) The graphs of f and f^{-1} intersect at point C. Find the coordinates of C. [3]

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6. [Maximum mark: 6]

Show that $\log_2 \sqrt{8} + \log_b \sqrt{ab} = \frac{\ln(ab^4)}{\ln(b^2)}$

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7. [Maximum mark: 6]

When the expression $3x^3 - 6x^2 + ax - 1$ is divided by $(x + 1)$ it produces the same remainder as when the expression is divided by $(x - 3)$. Find the value of a .

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8. [Maximum mark: 7]

Consider the following system of linear equations representing three planes.
A solution to the system is an ordered triple (x, y, z) – a point in space.

$$2x - y + 2z = 1$$

$$x + y - 2z = 2$$

$$x - 2y + 4z = -1$$

(a) Show, with justification, that the system has an infinite number of solutions. [4]

(b) Find the equation of the line, in parametric form, on which the solutions lie. [3]

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9. [Maximum mark: 7]

Find the value of $\int_0^{\frac{1}{2}} \frac{14x+1}{2x^2-x-1} dx$ giving your answer in the form $a \ln b$ where $a, b \in \mathbb{Z}$.

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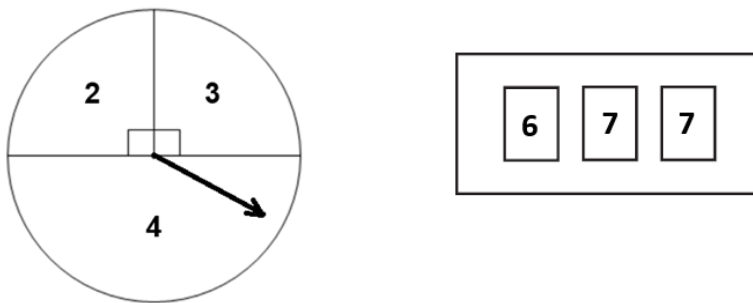
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Section B (54 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 20]

A spinner consists of an arrow that rotates about the centre of a circle so that one of three numbers is randomly selected (see diagram below). There is also a box containing three numbered cards as shown below. S is the sum of two numbers – one selected randomly with the spinner and the other from randomly selecting one of the cards from the box.



- (a) Write down the four different possible values of S . [2]
- (b) Find the probability of each value of S . [5]
- (c) Show that the expected value of S is $\frac{119}{12}$. [2]
- (d) Anna plays a game where she wins \$15 if S is an even number and loses \$10 if S is an odd number. Sophie plays the game 12 times. Find the amount of money she expects to have at the end of the 12 games. [6]
- (e) Anna now plays a different game where she wins \$14 if S is an even number. She loses x dollars if S is an odd number. Find the value of x so that the game is fair. [5]

11. [Maximum mark: 16]

- (a) Given that $(x + iy)^2 = 7 - 24i$, $x, y \in \mathbb{R}$, show that
- (i) $x^2 - y^2 = 7$;
- (ii) $xy = -12$; [4]
- (b) Hence, find the two square roots of $7 - 24i$ and express them in Cartesian form. [6]
- (c) For any complex number, show that $(z^*)^2 = (z^2)^*$. [4]
- (d) Hence, write down the two square roots of $7 + 24i$ in Cartesian form. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Consider the rational function $f(x) = \frac{1}{x-1}$.

(a) The notation $f^{(n)}(x)$ represents the n th derivative of f . Show that $f^{(3)}(x) = \frac{-6}{(x-1)^4}$. [4]

(b) Using mathematical induction, prove that the n th derivative of $\frac{1}{x-1}$ is $\frac{(-1)^n n!}{(x-1)^{n+1}}$. That is,

prove the statement $f^{(n)}(x) = \frac{(-1)^n n!}{(x-1)^{n+1}}$. [8]

Consider a different rational function $g(x) = \frac{x}{x-1}$.

(c) Prove that the graph of g can be produced by translating the graph of f one unit vertically in the positive y direction. [3]

(d) Hence, write down the formula for the n th derivative of g . [1]

The graph of g is symmetric about the line $y = x$.

(e) Deduce a statement regarding the function $y = g(x)$ and its inverse $y = g^{-1}(x)$. [2]

